

Abstract:

This is essentially the same proof that can be found on my website <http://donblazys.com/> and on the "Unsolved Problems Solutions Page" http://unsolvedproblems.org/index_files/Solutions.htm. Since it was released in January of 1999, I have received many requests to present it in the language of "first order logic" or "predicate calculus". This version makes use of standard logic and mathematical symbols and was prepared for those who prefer them over words.

Proof:

$$\begin{aligned} & \left[[(a^x + b^y = c^z) : (a, b, c, x, y, z \in \mathbb{N}) \wedge [(a \perp b), (a \perp c), (b \perp c)]] \right] \\ \leftrightarrow & [(x \leq 2) \vee (y \leq 2) \vee (z \leq 2)] \quad \neq \end{aligned}$$

$$\begin{aligned} \rightarrow & \left[\left[a^x + b^y = \sqrt{((b^y - a^x)^2 + 4a^x b^y)} = \sqrt{(c^z)^2} = c^z = \left(\frac{T}{T}\right) c^z = T \left(\frac{c}{T}\right)^{\frac{(z) \ln(c) - 1}{\ln(T)}} \right] \right. \\ \oplus & \left. \left[a^x + b^y = \sqrt{((b^y - a^x)^2 + 4a^x b^y)} = \sqrt{(c^z)^2} = (c^{\frac{z}{2}})^2 = \left(\left(\frac{T}{T}\right) c^{\frac{z}{2}}\right)^2 = \left(T \left(\frac{c}{T}\right)^{\frac{(\frac{z}{2}) \ln(c) - 1}{\ln(T)}}\right)^2 \right] \right] \end{aligned}$$

$$: [(x, y, z > 2), [(T > 1) \in \mathbb{N}]] \leftrightarrow (T \neq c) \rightarrow \left[\left(\frac{T}{T} = \frac{c}{c}\right) \wedge \left(\frac{T}{T} \neq \frac{c}{c}\right) \right].$$

$$\begin{aligned} \therefore & \left[[(a^x + b^y = c^z) : (a, b, c, x, y, z \in \mathbb{N}) \wedge [(a \perp b), (a \perp c), (b \perp c)]] \right] \\ \leftrightarrow & [(x \leq 2) \vee (y \leq 2) \vee (z \leq 2)] \quad \neq \end{aligned}$$

$$\therefore [(z = 1) \oplus (z = 2)]$$

$$\rightarrow \left[\left[a^x + b^y = \sqrt{((b^y - a^x)^2 + 4a^x b^y)} = \sqrt{(c^1)^2} = c^1 = \left(\frac{T}{T}\right) c^1 = T \left(\frac{c}{T}\right)^1 \right] \right]$$

$$\oplus \left[\left[a^x + b^y = \sqrt{((b^y - a^x)^2 + 4a^x b^y)} = \sqrt{(c^2)^2} = (c^1)^2 = \left(\left(\frac{T}{T}\right) c^1\right)^2 = \left(T \left(\frac{c}{T}\right)^1\right)^2 \right] \right]$$

$$: (T = c) \rightarrow \left(\frac{T}{T} = \frac{c}{c}\right). \quad \blacksquare$$

Notes:

The logarithmic identities can be similarly derived as follows:

$$\left(\frac{T}{T}\right) c^z = T \left(\frac{c}{T}\right)^{\frac{\ln\left(\frac{c^z}{T}\right)}{\ln\left(\frac{c}{T}\right)}} = T \left(\frac{c}{T}\right)^{\frac{\frac{\ln\left(\frac{c^z}{T}\right)}{\ln(T)} - \frac{\ln(T)}{\ln(T)}}{\frac{\ln(c)}{\ln(T)} - \frac{\ln(T)}{\ln(T)}}} = T \left(\frac{c}{T}\right)^{\frac{\frac{(z)\ln(c)}{\ln(T)} - 1}{\frac{\ln(c)}{\ln(T)} - 1}} .$$

The “Beal equation” can also be represented as:

$$\left[\left[c^z - b^y = \sqrt{((b^y + c^z)^2 - 4b^y c^z)} = \sqrt{(a^x)^2} = a^x = \left(\frac{T}{T}\right) a^x = T \left(\frac{a}{T}\right)^{\frac{\frac{(x)\ln(a)}{\ln(T)} - 1}{\frac{\ln(a)}{\ln(T)} - 1}} \right] \right]$$

$$\oplus \left[\left[c^z - b^y = \sqrt{((b^y + c^z)^2 - 4b^y c^z)} = \sqrt{(a^x)^2} = \left(a^{\frac{x}{z}}\right)^2 = \left(\left(\frac{T}{T}\right) a^{\frac{x}{z}}\right)^2 = \left(T \left(\frac{a}{T}\right)^{\frac{\frac{(x)\ln(a)}{\ln(T)} - 1}{\frac{\ln(a)}{\ln(T)} - 1}}\right)^2 \right] \right]$$

which demonstrates that *any* sum *or* difference of two terms is implicitly a square under a second degree radical and can therefore be used in a similar proof.

Meanings of Symbols:

\vDash	“Is true”	\wedge	“And”
\nVdash	“Is not true”	\vee	“Or” (One or the other or both.)
$>$	“Is greater than”	\oplus	“Or” (One or the other, but not both.)
\leq	“Is less than or equal to”	$:$	“Such that”
\rightarrow	“Implies” (If, then.)	\in	“Is an element of”
\leftrightarrow	“If and only if”	\perp	“Is co-prime to”
\therefore	“Therefore”	\mathbb{N}	“Natural Numbers”
\because	“Because”	\blacksquare	“End of proof”

Don Blazys.