A Special Polygonal Number Counting Function Involving The Fine Structure Constant. By: Don Blazys.

Abstract:

Polygonal numbers are among the most studied numbers in history, so it is important to ask how many of rank greater than $2$ there are under a given number $x$. In this paper, we introduce the function designated as $\mathcal{B}(x)$, which not only answers this question, but relates it to the ***fine structure constant***, thereby allowing a determination of its value by means that are strictly mathematical.

Article:

Let polygonal numbers of rank greater than $2$ be defined as the various different numbers:

$\mathcal{P}_{n>2}^{r>2} = 6, 9, 10, 12, 15, 16, 18, 21, 22, 24, 25, 27, 28, 30 \ldots$ which are generated by the formula:

$\left(\frac{n}{2}-1\right) * r^2 - \left(\frac{n}{2}-2\right) * r$, when integers $n$ and $r$ are greater than $2$,

and let $\varpi(x)$ represent how many such numbers there are less than or equal to a given number $x$.

Then, $\varpi(x) \sim \mathcal{B}(x) = x - (\alpha * \pi * e + e)^{-1} * x - \frac{1}{2} * \sqrt{x - (\alpha * \pi * e + e)^{-1} * x}$

where $\alpha \approx 137.03599908451^{-1}$ is the ***fine structure constant***.

The table below represents $\varpi(x)$ approximated by $\mathcal{B}(x)$:

| $x$ | $\varpi(x)$ | $\mathcal{B}(x)$ | Difference | Percentage of error |
|---|---|---|---|---|
| $10$ | $3$ | $5$ | $2$ | $.666666667$ |
| $100$ | $57$ | $60$ | $3$ | $.052631579$ |
| $1{,}000$ | $622$ | $628$ | $6$ | $.009646302$ |
| $10{,}000$ | $6{,}357$ | $6{,}364$ | $7$ | $.001101148$ |
| $100{,}000$ | $63{,}889$ | $63{,}910$ | $21$ | $.000172174$ |
| $1{,}000{,}000$ | $639{,}946$ | $639{,}965$ | $19$ | $.000029690$ |
| $10{,}000{,}000$ | $6{,}402{,}325$ | $6{,}402{,}388$ | $63$ | $.000009840$ |
| $100{,}000{,}000$ | $64{,}032{,}121$ | $64{,}032{,}528$ | $407$ | $.000006356$ |
| $1{,}000{,}000{,}000$ | $640{,}349{,}979$ | $640{,}352{,}643$ | $2664$ | $.000004160$ |

Note that $\mathcal{B}(x)$ is an upper bound with a very small and ever decreasing percentage of error. Clearly, this is extraordinary and compelling evidence that the fine structure constant is not only a "physical constant", but a *mathematical* constant as well, and that its value might now be determined by determining $\varpi(x)$ for increasingly large values of $x$, and adjusting $\alpha$ so that $\mathcal{B}(x)$ *remains* an upper bound with a very small and ever decreasing percentage of error. ■